of points increases at frequencies higher than 83 Hz, where more than three points from 11 at each frequency fall outside the confidence interval. The scatter increases also at frequencies higher than 170 Hz, where 4-6 points out of 11 lie outside the confidence interval. The results of calculation for the spectral density of the transverse velocity component at $2Re_{min}$ agree with the hypothesis of stationarity in the frequency region under study.

The experimental test of the statistical properties of velocity pulsations of a developed turbulent flow in a round pipe showed that in the region of developed turbulence for Re_{min}, one observes the nonstationarity of small-scale perturbations in the center of the flow. Near the wall, the nonstationarity manifests itself in the rms deviations of velocity pulsations. Thus, when considering the develped turbulence at small Reynolds numbers, one must take into account the statistical nonstationarity of pulsating velocity components.

NOTATION

$G_B(f_i)$, spectrum of the sample of estimates of the rms deviations; f, frequency; N, dimension of the sample of estimates of the rms deviations; Δt, discretization time of the process; $H(f_i)$, amplitude–frequency characteristic; τ_{max}, maximum correlation interval; U_0, flow rate velocity; n, number of neighboring frequencies; and m, number of realizations.

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CALCULATION OF TURBULENT GAS JETS

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A technique is proposed for calculating jets with variable density. The variations of the basic parameters of a jet are obtained and they are compared with experimental data.

In solving the problem analytically, the flow in a jet was taken as self-similar under the assumption that the external boundary and the streamlines in the main part of the jet are curvilinear and emanate from the terminal (Fig. 1). Based on experiments, it was assumed that the dimensionless profiles of excess temperatures, excess enthalpies and concentrations in the transverse cross sections of the jet coincide [1]:

$$\frac{\Delta T}{\Delta T_m}=\frac{\Delta i}{\Delta i_m}=\frac{C}{C_m}. \tag{1}$$

In the starting section of the jet, the dimensionless profiles of velocities and temperatures (enthalpies and concentrations) were determined with an accuracy useful for practical applications, from the equations

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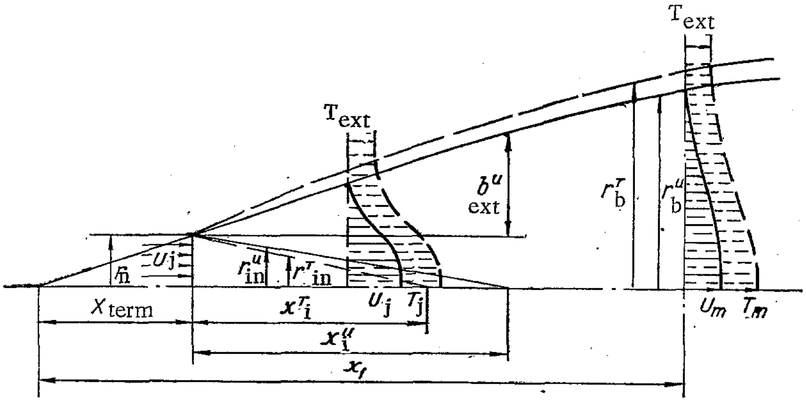

Fig. 1. Diagram of the jet.

$$\frac{u}{U_j} = 1 - (1 - \eta^{3/2})^2, \tag{2}$$

$$\frac{\Delta T}{\Delta T_j} = \frac{T - T_{ext}}{T_j - T_{ext}} = \frac{i - i_{ext}}{i_j - i_{ext}} = \frac{C}{C_j} = 1 - (1 - \eta_T^{3/2})^2, \tag{3}$$

where

$$\eta = \frac{r - r_b^u}{r_{in}^u - r_b^u}; \qquad \eta_T = \frac{r - r_b^T}{r_{in}^T - r_b^T}.$$

The distribution of velocities and excess temperatures (enthalpies and concentrations) in the transverse cross sections of the main part of the jet corresponds to the theoretical profiles, which can be represented in the form

$$\frac{u}{U_m} = (1 - \xi^{3/2})^2, \tag{4}$$

$$\Delta \overline{T} = \frac{\Delta T}{\Delta T_m} = \frac{T - T_{ext}}{T_m - T_{ext}} = \frac{i - i_{ext}}{i_m - i_{ext}} = \frac{C}{C_m} = [1 - (0.865\,\xi)^{3/2}]^2, \tag{5}$$

where $\xi = r/r_b^u$ and $r_b^u/r_b^T = 0.865$.

The conditions for conservation of momentum and excess enthalpy with uniform flow parameters at the nozzle edge

$$\int_0^F \rho u^2 dF = \rho_j U_j^2 F_n, \tag{6}$$

$$\int_0^F \rho u (i - i_{ext})\, dF = \rho_j u_j (i_j - i_{ext}) F_n \tag{7}$$

can be written as follows for an axisymmetrical jet:

$$\left(\frac{U_m}{U_j}\right)^2 \left(\frac{r_b^u}{X_1}\right)^2 \left(\frac{X_1}{r_n}\right)^2 A_2 = 1, \tag{8}$$

$$\frac{U_m}{U_j} \frac{\Delta i_m}{\Delta i_j} \left(\frac{r_b^u}{X_1}\right)^2 \left(\frac{X_1}{r_n}\right)^2 B_2 = 1, \tag{9}$$

where

$$A_2 = 2\int_0^1 \frac{\rho}{\rho_j} \left(\frac{U}{U_m}\right)^2 \xi d\xi; \; B_2 = 2\int_0^1 \frac{\rho}{\rho_j} \frac{U}{U_m} \frac{\Delta i}{\Delta i_m} \xi d\xi.$$

For an isobaric mixing process, using Dalton's law, the ratio of the density of the mixture to the density of the jet at the nozzle edge in the expressions for A_2 and B_2 was determined according to the equation of state:

$$\frac{\rho}{\rho_j}=\frac{T_j}{T}\,\frac{\dfrac{\mu_{ext}}{\mu_j}}{C\dfrac{\mu_{ext}}{\mu_j}+(1+C)}. \tag{10}$$

For zero admixture concentration, taking into account Eq. (1), we shall write (10) as

$$\frac{\rho}{\rho_j}=\frac{T_j}{T}=\frac{\dfrac{T_j}{T_{ext}}}{1+\left(\dfrac{T_j}{T_{ext}}-1\right)\Delta\overline{T}\Delta\overline{T}_m}, \tag{11}$$

where

$$\Delta\overline{T}_m=\frac{\Delta T_m}{\Delta T_j}=\frac{T_m-T_{ext}}{T_j-T_{ext}}=\frac{i_m-i_{ext}}{i_j-i_{ext}}=\frac{C_m}{C_j}.$$

For jets with a molecular weight different from that of air and $T_j = T_{ext}$, Eq. (10) taking into account (1) takes the form

$$\frac{\rho}{\rho_j}=\frac{\dfrac{\mu_{ext}}{\mu_j}}{1+\left(\dfrac{\mu_{ext}}{\mu_j}-1\right)\Delta\overline{T}\Delta\overline{T}_m}. \tag{12}$$

In a plasma jet with water stabilization, the following relation was used between the density and enthalpy [2]:

$$\frac{\rho}{\rho_j}=\left(\frac{i_j}{i}\right)^{0,91}, \tag{13}$$

which, taking into account Eq. (1), can be written in the form

$$\frac{\rho}{\rho_j}=\left[\frac{\dfrac{i_j}{i_{ext}}}{1+\left(\dfrac{i_j}{i_{ext}}-1\right)\Delta\overline{T}\Delta\overline{T}_m}\right]^{0,91}. \tag{14}$$

Writing down the equation of state in this form permitted calculating quite simply the value of the density of the mixture at any point in the jet from Eqs. (11), (12), and (14), as well as the integrals A_2 and B_2 as a function of the parameters T_j/T_{ext}, i_j/i_{ext}, and μ_{ext}/μ_j with $\Delta\overline{T}_m$ varying from unity to zero. The parameters in various regimes and the notation for jets studied are presented in Table 1.

TABLE 1. Jet Parameters in Various Regimes

Working body of jet	r_n, mm	x, mm	$\frac{T_j}{T_{ext}}$	$\frac{\mu_{ext}}{\mu_j}$	$\frac{\rho_j}{\rho_{ext}}$	Conventional notation	From data in reference number
Freon	2,5	70—145	1	0,313	3,2	I	[7]
Air	5,0	50—150	1	1,0	1,0	II	[2]
Air	45	600—1600	1	1,0	1,0	III	[1]
Helium	2,5	70—145	1	8,0	0,125	IV	[7]
Air	15,0	250—500	15	1,0	0,067	V	[9]
Plasma	2,5	25—40	$\frac{i_j}{i_{ext}}=440$	—	0,004	VI	[2]

TABLE 2. Parameters in Transverse Cross Sections of Jets

ξ	$\frac{u}{U_m}$	$\Delta\bar{T}$	$\Delta\bar{T}_m$						
			0	0,1	0,2	0,4	0,6	0,8	1,0
			ρ/ρ_j						
0	1,0	1,0	15	6,25	3,95	2,27	1,6	1,23	1,0
0,2	0,83	0,865	15	6,8	4,38	2,6	1,82	1,41	1,15
0,4	0,56	0,632	15	7,96	5,39	3,29	2,38	1,85	1,52
0,6	0,29	0,392	15	9,7	7,12	4,7	3,5	2,78	2,3
0,8	0,08	0,18	15	12,0	10,0	7,42	5,98	5,0	4,3
1,0	0	0,038	15	14,2	13,5	12,3	11,3	10,5	9,8
1,156	—	0	15	15,0	15,0	15,0	15,0	15,0	15,0

Table 2 gives the dimensionless values of the velocities u/U_m and temperatures $\Delta\bar{T}$, as well as the relative density ρ/ρ_j in the transverse cross sections of the principal part of the jet for different $\Delta\bar{T}_m$ with initial heating of the jets $T_j/T_{ext} = 15$.

Dividing Eq. (8) by (9), we obtain the following relation between the axial velocity of the jet and the temperature $\Delta\bar{T}_m$:

$$\frac{U_m}{U_j} = \frac{B_2}{A_2}\Delta\bar{T}_m. \tag{15}$$

In order to find the drop in the velocity U_m/U_j and excess temperature $\Delta\bar{T}_m$ along the jet from Eqs. (8) and (9), it is necessary to know the change in r_b^u/X_1, characterizing the expansion of the jet along its length.

The coefficient of expansion of the jet C in the principal part of a jet of incompressible liquid ($\bar{\rho} = \rho_j/\rho_{ext} = 1$), according to data in [1] and elsewhere, was taken as equal to 0.22. However, a more thorough analysis of the experimental data showed that the coefficient C with $\bar{\rho} = 1$ equal 0.167 [3, 4]. It is known from experiments that the coefficient of expansion of the external boundary in the initial section $C_{ext}^u = b_{ext}^u/X_1^u$ increases with decreasing density of the jet [3, 4]. At large distance from the nozzle (for $\rho_m = \rho_{ext}$), independent of the density of the jet at the outlet from the nozzle, the coefficient of expansion of the jet becomes equal to 0.167. In addition, for jets whose density is less than the density of the surrounding medium ($\rho_j > \rho_{ext}$), the coefficient C must decrease with distance from the nozzle, while for jets with $\rho_j > \rho_{ext}$, it must increase. For jets of incompressible liquid ($\rho_j = \rho_{ext}$), the coefficient C remains constant along the jet.

Based on what was said above, the following empirical law is proposed for the coefficient of expansion of the jet:

$$C = \frac{r_b^u}{X_1} = C_{\bar{\rho}=1}\left[1.634 - 0.317\left(\lg 100\,\frac{\rho_m}{\rho_{ext}}\right)\right]. \tag{16}$$

The relative density on the axis of the jet ρ_m/ρ_{ext} for homogeneous jets was determined from Eq. (11) with $\Delta\bar{T} = 1$:

$$\frac{\rho_m}{\rho_{ext}} = \frac{1}{1+\left(\frac{T_j}{T_{ext}} - 1\right)\Delta\bar{T}_m}. \tag{17}$$

For jets whose composition differs from that of air, from Eq. (12):

$$\frac{\rho_m}{\rho_{ext}} = \frac{1}{1+\left(\frac{\mu_{ext}}{\mu_j} - 1\right)\Delta\bar{T}_m}. \tag{18}$$

For plasma jets, from Eq. (14):

$$\frac{\rho_m}{\rho_{ext}} = \frac{1}{\left[1+\left(\frac{i_j}{i_{ext}} - 1\right)\Delta\bar{T}_m\right)^{0,91}}. \tag{19}$$

TABLE 3. Variation in the Parameters along a Jet

$\Delta\overline{T}_m$	ρ_j/ρ_{ext}														
	0,004			0,067			0,125			1,0			3,2		
	A_2	B_2/A_2	C	A_2	B_2/A_2	C	A_2	B_2/A_2	C	A_2	B_2/A_2	C	A_2	B_2/A_2	C
0	34,5	1,135	0,167	1,98	1,135	0,167	1,08	1,135	0,167	0,135	1,135	0,167	0,0424	1,135	0,167
0,1	1,61	1,19	0,247	1,034	1,16	0,187	0,73	1,149	0,179	0,135	1,135	0,167	0,0445	1,134	0,165
0,2	0,869	1,196	0,261	0,702	1,17	0,198	0,556	1,157	0,187	0,135	1,135	0,167	0,047	1,131	0,164
0,4	0,467	1,196	0,275	0,433	1,18	0,212	0,378	1,169	0,198	0,135	1,135	0,167	0,053	1,127	0,160
0,6	0,324	1,197	0,285	0,317	1,186	0,2195	0,288	1,175	0,205	0,135	1,135	0,167	0,06	1,121	0,155
0,8	0,25	1,197	0,29	0,245	1,19	0,225	0,232	1,182	0,21	0,135	1,135	0,167	0,071	1,114	0,149
1,0	0,204	1,197	0,294	0,198	1,192	0,230	0,195	1,183	0,215	0,135	1,135	0,167	0,088	1,105	0,140

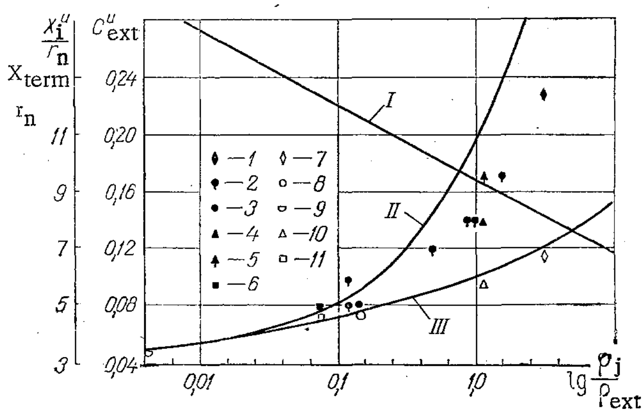

Fig. 2. I) Change in the coefficient of expansion of a jet C^u_{ext}; II) length of the initial section X^u_i/r_n (1-3, [7]; 4, [2]; 5, [1]; 6, [9]); III) distance from the nozzle edge to the pole X_{term}/r_n (7, 8, [7]; 9, 10, [2]; 11, [9]) as a function of ρ_j/ρ_{ext} (the curves indicate the calculation).

The values of the integrals A_2, B_2/A_2, and the coefficient of expansion of the jet C for different parameters ρ_j/ρ_{ext} and $\Delta\overline{T}_m$ are presented in Table 3.

In the starting section of the jet, the coefficient of expansion along the outer boundary was determined from Eq. (16) by replacing ρ_m by ρ_j:

$$C^u_{ext} = \frac{b^u_{ext}}{X^u_i} = C_{\bar{\rho}=1}\left[1.634 - 0,317\left(\lg 100\,\frac{\rho_j}{\rho_{ext}}\right)\right]. \tag{20}$$

The length of the starting section of the jet was found from Eq. (8) with $U_m = U_j$:

$$\left(\frac{X^u_i}{r_n}\right)_i = \frac{1}{C^u_{ext}\sqrt{A_2}}. \tag{21}$$

The distance from the nozzle edge to the terminal was determined from the geometric characteristics of jets (see Fig. 1).

$$\frac{X_{term}}{r_n} = \frac{1}{C^u_{ext}}. \tag{22}$$

The length of the starting section according to the temperature can be found from Eq. (9), if it is assumed that $\Delta\overline{T}_m = \Delta\overline{T}_j$ and the value of B_2 at $\Delta\overline{T}_m = 1$ is

$$\left(\frac{X^T_i}{r_n}\right)_i = \frac{1}{C^u_{ext}\sqrt{B_2}}. \tag{23}$$

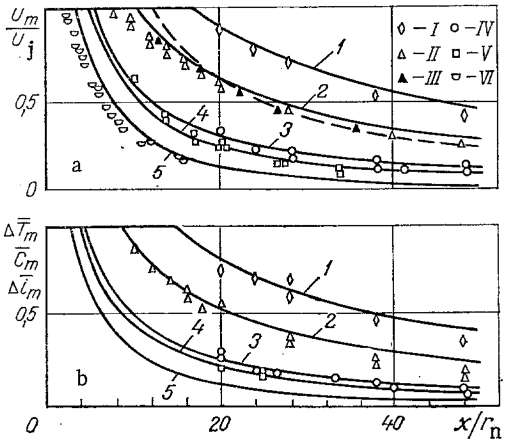

Fig. 3. a) Change in the relative velocity [1) $\rho_j/\rho_{ext} = 3.2$; 2) 1; 3) 0.125; 4) 0.067; 5) 0.004]; b) change in the excess concentration [1, 3) 3.2; 0.125], excess temperature [2, 4) 1; 0.067] and enthalpy [5) $\rho_j/\rho_{ext} = 0.004$] along the jet axis (the curves show the calculation); the points I-VI are taken from Table 1.

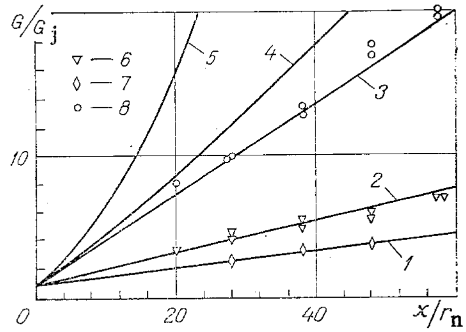

Fig. 4. The variation of the relative added mass along the jet: 1) ρ_j/ρ_{ext} = 3.2; 2) 1; 3) 0.125; 4) 0.067; 5) ρ_j/ρ_{ext} = 0.004 (6-8, [7]).

As is evident from Fig. 2, when the density of the jet increases, the length of the starting section and the distance to the pole increase, while the coefficient of expansion of the jet drops. This change in the quantities indicated qualitatively agrees with available experimental data [2-9]. The difference between the experimental and computed data on the length of the initial section is related to the fact that the length of the starting section was determined without taking into account the length of the transition section. Figure 3 shows the change in the parameters along the axis of the jet: the lines show the change in the velocities, found from Eq. (8), and the change in temperatures, enthalpies, and concentrations from Eq. (9). It is evident from the graphs that the experimental and computed data agree satisfactorily. A decrease in the density of the jet leads to a more intense drop in its axial parameters and decreases the length of the initial section. If it is assumed, as done by Tryupel [1], that for a jet of incompressible liquid, the coefficient of expansion of the jet C = 0.22 and the terminal of the jet is placed at the nozzle edge, then from the conservation of momentum (8) the length of the initial section $X_i^u/r_n = 12.4$ and the drop in the velocity along the jet, found from Eq. (8) (dashed curve), do not correspond at short distances from the nozzle to well-known experimental data, including also Tryupel's data. The change in the added mass along the jet can be determined by writing the flow rate in an arbitrary cross section and at the nozzle edge as

$$G = \int_0^F \rho u dF, \quad G_j = \rho_j U_j F_n. \tag{24}$$

As a result of simple transformations, we obtain

$$\frac{G}{G_j} = \frac{U_m}{U_j}\left(\frac{r_b^u}{X_1}\right)^2 \left(\frac{X_1}{r_n}\right)^2 A_1, \quad (25)$$

where

$$A_1 = 2\int_0^1 \frac{\rho}{\rho_j} \frac{U}{U_m} \xi d\xi.$$

The variation of the added mass and comparison between experimental and computed data are shown in Fig. 4. It should be noted that the experimental data in [10] exceed the results of the present work by 20-25%. This disagreement can be explained by the fact that in [10] the added mass was determined by a different method and for this reason it is difficult to compare these data.

NOTATION

X, instantaneous coordinate along the axis of the jet; r_n and F_n, radius and area of the nozzle; U_j, velocity of the jet at the nozzle outlet; ρ_j and ρ_{ext}, densities of jet and surrounding medium; μ_{ext}, molecular weights of the jet and of air; C, weight concentration of the admixture; i, enthalpy; ξ and η, independent dimensionless variables; F, area of the transverse section of a jet.

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